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AUTHOR Deal, Therry N.
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ABSTRACT

This study reports a factor analysis of the data resulting from a 3-year longitudinal investigation into conservation of number and related mathematical type concepts. Conservation of number, which represents a subset within the concept of conservation, was measured by use of a 20-item, criterion-referenced testing device. The subjects, who were tested 3 years in succession during a 4-week period each spring, were 5, 6, and 7 years of age (at the time of the last data collection). The factor analysis of the test results revealed a pattern of development that showed an increasing emergence of the conservation factor. Furthermore, successive factor analyses indicated that this conservation factor has its roots in related subskills, particularly vocabulary discrimination. The overall developmental pattern, as shown by this analysis, moves from diffuseness toward increasing organization as the subskills develop and conservation emerges. These results are seen as support for Piaget's theoretical notion of the emergency of conservation. (MH)

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A Factor Analysis of a Three-Year Longitudinal Study of Conservation of Number and Related Mathematical Concepts[1]

Therry N. Deal
University of Georgia

Factor-analytic techniques were applied to longitudinal data from a criterion-referenced test which measured conservation of number and other mathematical type content. A factor for comparative-superlative vocabulary was revealed. A developmental pattern describing sub-skills of conservation of number emerged.

Conservation has been considered a most important index in determining when children move from preoperational to concrete operational thought. Conservation of number represents a subset within the concept of conservation. Piaget (1952) has linked numerical thinking with logical thinking in general and has attempted to show that their development is concomitant. A major problem in research has been to determine the subskills prior to conservation (Gruen, Zimiles, 1966). It is of particular significance to know these if we wish to accurately describe preoperational thinking; it is likewise important to know these if we wish to design curricula which will accelerate or least adequately provide experiences needed in the development of these skills.

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In spite of the tremendous volume of work done with reference to conservation, as yet no detailed appraisal of the course of its development has emerged (Sigel & Hooper, 1968). Zimiles's (1966) words ring true today:

> . . . careful study of any cognitive performance emphasizes its complexity. It would seem more realistic to conduct functional analysis of measures in order to identify their underlying net work of cognitive processes than to see samples of behavior that are themselves irreducible cognitive elements . . . It is the organizing and integrating aspect of cognition that needs to be delineated if complex forms of behavior are to be understood."

The emergence of the criterion-referenced test (Popham & Husek, 1969) where an individual's status is determined with reference to a standard has relevance to educators and developmentalists who are more concerned with order of change than normative information. Factor analytic techniques have been used most ingeniously by Cattell (1967) and Damarin and Catell (1968) in attempting to determine the developmental structural patterns of both intelligence and personality. These studies suggest the possible application of factor-analysis to longitudinal data rather than cross-sectional data in an effort to delineate the structure of cognition in an empirical fashion.

Objective

The purpose of this study was to describe longitudinally the factor structure of a measure designed to tap conservation of number and related mathematical type concepts.

Subjects

The data reported in this research were obtained from 130 subjects in a demonstration center designed to study the longitudinal effects of early and continuous educational stimulation. The subjects in the center were selected to approximate certain demographic variables in the region in which the center is located.

Subjects were 167 children, three, four and five years of age at the first data collection; 145 children four, five and six years of age at the second collection and 130 children five, six and seven years of age at the third collection period. The same subjects were measured on each occasion. The change in size of N represents subjects lost from the sample during the three-year period.

Methods

The testing device used for data collection contained twenty items designed to tap certain vocabulary, matching and conservation of number abilities. An outline of the test items appears in Figure 1. More detailed analysis of the test are available in previous research (Deal 1967, 1969a, 1969b).

The data reported were collected three years in succession during a four-week testing period each spring. Three graduate research assistants obtained the data in both 1968 and 1969.

Data Analysis

Separate 20 x 20 tetrachoric matrices were obtained for each test year. The analysis of the first year data using iterative procedures indicated five first order factors. These were rotated using a Varimax procedure (Wright, 1963). A five-factor solution was obtained for the following two years in order to provide for comparability of the data.

Results

The results are based on the rotated factors. All factor loadings of .20 and above are indicated in Table I. Four factors in the first year accounted for 85% of communality. Four factors accounted for 89% of the communality in the second year. In the third year, three factors represented 83% of the communality.

An orderly picture of easy and difficult items emerged. Two items were easy in the first year, seven in the second and ten items in the third. With few exception items remained stable when they once occurred as easy. Four items were difficult in the first year; one the second; and none the third year.

In the first year analysis the percentage of communality was distributed with approximate equivalence across all five factors. The percentages were 27, 21, 19, 19 and 15 respectively (Figure 2). Factor I and Factor II involved discrimination of aggregates of number and labeling with comparative or superlative forms of speech.

Items 12, 13, 14 and 18 are the items containing standard conservation tasks. During the first year the conservation items formed a

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part of Factors III and IV. Moderate positive correlations for items 13 and 14 also had high negative loadings on Factor IV; this factor also included item 12 where the task was recognition of equivalence. Factor V was labeled identification using motor skills.

In the second year analysis items 12 and 13 alone form Factor II; this suggested the emergence of a weak conservation factor. Factor III, labeled perceptual - configuration contained item 14, a standard conservation task in which objects appear in subsets. Comparative vocabulary was the label given to Factor I. The fourth factor was composed of easy items.

The analysis of the third year revealed a clear conservation factor including items 13, 14 and 18. Factor I was specific to the vocabulary of less and least. Factor III, labeled perceptual equivalence, accounted for only 16% of the communality. Factors I and II jointly accounted for 67%.

Discussion

Conservation of number is the single dimension of the operation of conservation considered in this paper. This discussion must also be viewed within the context of the population of items included in the analysis. With these limitations in mind, a model is proposed to represent the development of conservation and the sub-skills which are its roots as revealed in the successive factor analyses. A schematic presentation of the developmental pattern is presented in Figure 2. Note that first-, second- and third-year analyses are assumed to represent

developmental change from younger to older and less experienced to more experienced children.

The overall pattern moves from diffuseness toward increasing organization. Within the general organization two clearly discernable factors emerge. One of these deals with comparative-superlative vocabulary discrimination. The other is a dimension of conservation.

Level one represents the initial or basic level of response indexed by the tasks used in the study. Factor V, identification using motor skills, appears to be on the wane. We can assume that it is the two- and three-year-old child who is likely to need emphasis upon this. Conservation tasks appear in conjunction with two other type tasks. Factor IV, included a conservation item, but the central meaning of the factor revolves around establishing equivalency. Transformation tasks are negatively related to equivalency in this factor. The teaching implication would be to provide the child many experiences in matching sets. The other root of conservation, Factor III, involves perceptual discriminations to which a vocabulary label is simultaneously attached. Experiences in attending to shape and arrangement of materials would presumably facilitate this skill. Vocabulary learning is apparently pervasive at this level as vocabulary items are found in Factors I, II, and III. This emphasizes the need for much opportunity to talk about experiences.

At the second level a definite vocabulary factor including most of the comparative and superlative vocabulary item is present. Teaching strategies at this point would involve opportunities for increase in

precision of descritive word use. The conservation factor is weak or transitional; minimal transformations are recognized but more extensive ones are not. Continued learning opportunities involving experience in perceptual arrangements of varying types as demonstrated by connection of conservation items to this factor are still needed. Here appears to be the time to provide direct experiences in transformation tasks.

By the time the third level is reached the factor pattern suggests that superlative forms of vocabulary are independent of conservation. On one hand this seems in keeping with Piaget's claim that language is not the basis for thought structures. On the other hand, superlative forms of speech appear to provide a verbal symbolic representation of seriation. Piaget has indicated seriation to be concomitant with conservation. On closer analysis of the tasks in the factors these findings do not indicate that perceptual-motor seriation is not or has not been important in the development of conservation. The presence of the "most" items at an earlier level suggests that perceptual seriation is important but that minimal vocabulary for it may be sufficient as the basis is operational itself. That is, if one has the concept of most, he can manage conservation without the verbal symbol least.

The factor pattern at third level also suggests the interpretation that when conservation is well established new emphasis on vocabulary involving increasingly sophisticated denotation is implied. At level three the child has reached concreats operations as indexed by number conservation. Emphasis on word skill is congruent with Piaget's emphasis on formation of alternative hypotheses at the concrete level.

A mode for explicit formulation must be available and language is one set of symbols which provides this mode. Language then is never unimportant; this interpretation simply supports the position that it is not a single causative variable.

Summary

Empirical support for Piaget's theoretical notion of the emergence of conservation appeared in the increasingly clear factor for conservation of number. A picture of sub-skills required for conservation was also suggested. Implications for teaching were drawn from this.

When all of the children in the study were under six years CA, the item picture was clearly that of diffuseness. With increasing maturity the organization of the factors becomes much more cohensive and it is possible to locate factors more definitely. These results indicate that the meaning of the same task changed for the same children with increasing age and developmental maturity.

As Zimiles (1966) suggested, psychologists tend to dichotomize children's concepts as present or absent. He contends and this author concurs that concepts are not neatly compartmentalized with all contraditions eliminated. Longitudinal factor-analysis of the same tasks appears to be a useful technique for revealing these contraditions and the circuitous route of their development.

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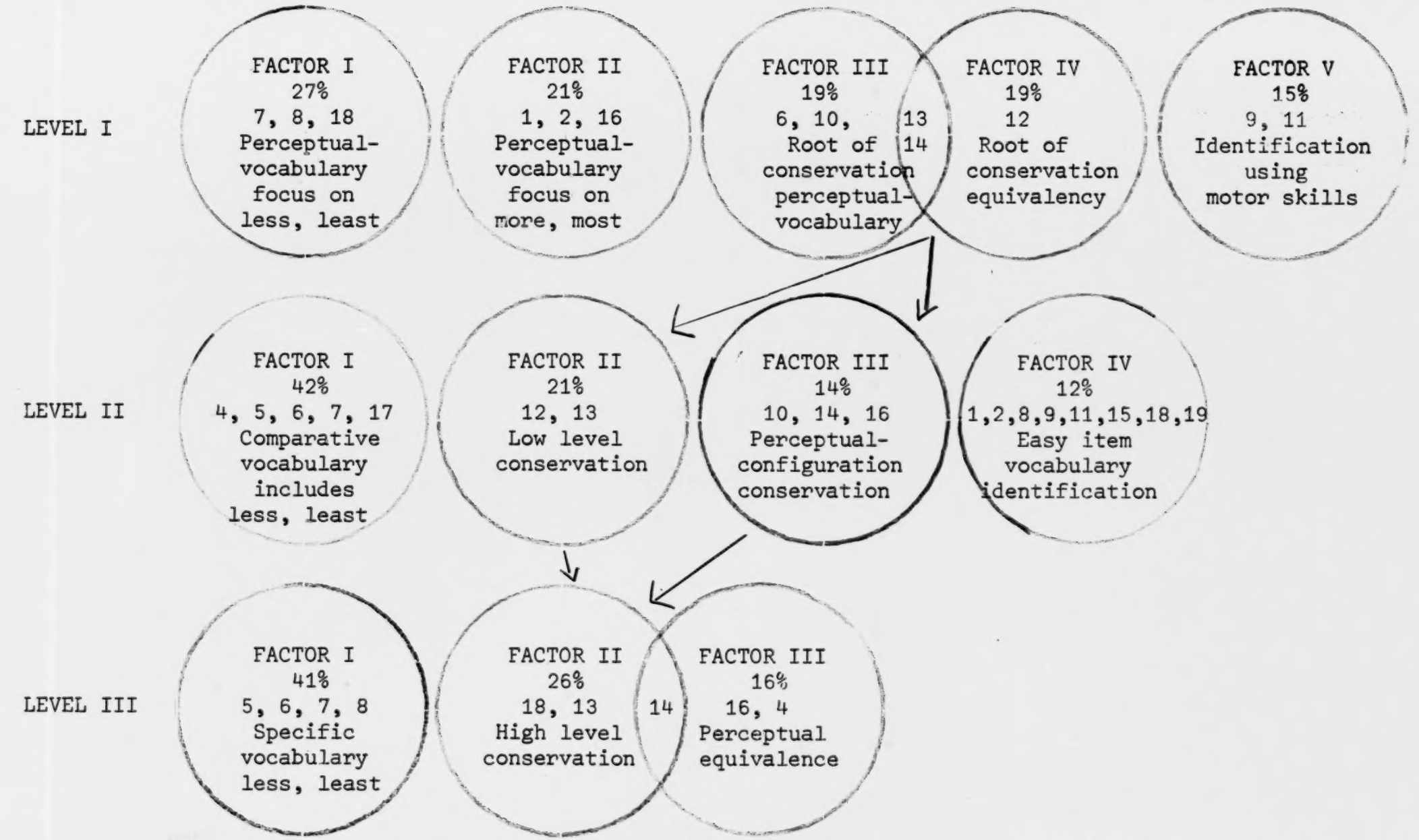

Note: % in each circle represents percentage of communality contributed by factor.

FIGURE 2

EMPIRICAL DEVELOPMENT OF SUB-SKILLS OF CONSERVATOPM
AND COMPARATIVE AND SUPERLATIVE VOCABULARY

TABLE I

Factor Loadings for Three-Year Comparison

	1967 (N = 167)						1968 (N = 145)						1969 (N = 130)					
	Factors						Factors						Factors					
ITEM	a	I	II	III	IV	V	a	I	II	III	IV	V	a	I	II	III	IV	V
1.			.66				easy				-.26		easy					
2.			.60								-.24	-.38	easy					
3.	easy											-.39	easy					
4.	easy							.89									.36	
5.								.74				-.24		.48				
6.				.50				.87						.69				.34
7.		.81						.87						.64				-.20
8.		.83					easy				-.25			.79				
9.						.59	easy				-.25		easy					
10.			.67				easy			.39			easy					
11.					.59		easy				-.31		easy					
12.				.36					.86				easy					
13.			.48	-.63					.83						.65	-.25		
14.			.27	-.71						.66					.67		.24	
15.	diff.						easy				-.32					.53		
16.		.49								.60			easy				.32	
17.	diff.							.36				-.47				.52		
18.		.37					easy				-.38				.48	.23		
19.	diff.										-.30		easy					
20.	diff.						diff.						easy					

a. Item analysis indicates this item should be eliminated for reason indicated.
b. Only item with no loading of .20 NOT marked easy or difficult.

I. Vocabulary

1. Child asked to identify card with "more" dogs - 2 cards: 5 dogs; 12 dogs.
2. Child asked to identify card with "most" black things 3 cards: 2, 5, 12 items resp.
3. Child asked to identify card with "more" black things 2 cards: 5, 6 items resp.
4. Child asked to identify card with "most" dogs, 3 cards: 5, 6, 7 items resp.
5. Child asked to identify card with "less" black things 2 cards 12, 5 items resp.
6. Child asked to identify card with "least" black things 3 cards: 2, 5, 12 items resp.
7. Child asked to identify card with "less" dogs 2 cards: 5, 6 items resp.
8. Child asked to identify card with "least" dogs 3 cards, 6, 7, 5 items resp.
9. Child asked to point to triangle.
10. Child asked to point to square.

II. Operations

11. Child asked to establish equivalence by putting eggs into cups
12. Child asked to ascertain equivalence of eggs & egg cups by observation
13. Child asked to ascertain equivalence by observation under condition of transformation - (eggs were extended)
14. Child asked to ascertain equivalence by observation under condition of transformation - (eggs broken into subsets)
15. Child asked to establish equivalence (1 bone for 1 dog)
16. Child asked to ascertain equivalence of dog & bone by observation
17. Child asked to establish equivalence of chips by placing another row (7 chips)
18. Child asked to ascertain non-equivalence when line of 6 chips extended so that it was longer than line of 7 chips
19. Child asked to ascertain equivalence of chips in 1 to 1 correspondence (7 items)
20. Child asked to ascertain equivalence of chips in 1 to 1 correspondence (12 items)

FIGURE 1

OUTLINE OF TEST ITEMS